Non-Rigid Designators in Epistemic and Temporal Free Description Logics (Extended Abstract)

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Abstract
Definite descriptions, along with individual names, have been recently introduced in the context of description logic languages, enriching the expressivity of standard nominal constructors. Moreover, in the first-order modal logic literature, definite descriptions have been widely investigated for their *non-rigid* behaviour, which allows them to denote different objects at different states. In this direction, we introduce *epistemic* and *temporal* extensions of standard description logics, with nominals and the universal role, additionally equipped with definite descriptions constructors. In the absence of the rigid designator assumption, we show that the satisfiability problem for epistemic free description logics is NExpTime-complete, while satisfiability for temporal free description logics over linear time structures is undecidable.

Keywords
Epistemic and temporal description logics, Definite descriptions, Non-rigid designators

1. Introduction

Definite descriptions, like 'the smallest planet in the Solar System', are expressions having form 'the x such that φ'. Together with *individual names*, such as 'Mercury', they are used as *referring expressions* to identify objects in a given domain [1, 2, 3]. Definite description and individual names can also *fail to denote* any object at all, as in the cases of the definite description 'the planet between Mercury and the Sun' or the individual name 'Vulcan'. Formal accounts that address these aspects and still admit definite descriptions as genuine terms of the language, on a par with individual names, are usually based on so-called *free logics* [4, 5, 6, 7]. These are in contrast with classical logic approaches, in which individual names are assumed to always designate, and where definite descriptions are paraphrased in terms of sentences expressing existence and uniqueness conditions (an approach dating back to Russell [8]). Recently, definite descriptions have been introduced into description logic (DL) formalisms [9, 10, 11] as well.

In *modal* settings, such as temporal or epistemic, referring expressions can also behave as *non-rigid designators*, meaning that they can denote different individuals across different states (epistemic alternatives, instants of time, etc.). For this reason, non-rigid descriptions and names

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have been widely investigated in first-order modal and temporal logics [12, 13, 14, 15, 16, 17, 18, 19]. However, with the exception of [20], non-rigid designators have received little attention in modal DLs, despite the interest in temporal [21, 22, 23] and epistemic [24, 25, 26] extensions.

In this paper, we extend the free DLs proposed for the non-modal case in [10, 11], by: (i) adding *epistemic modalities*, such as $\Box$ (*box*, read as 'it is known that'), or *temporal* ones, like $\mathcal{U}$ (*until*); (ii) introducing nominals built from *definite descriptions* of the form ιC (read as 'the object that is C'), where C is a concept, alongside the standard ones based on *individual names*; (iii) dropping the *rigid designator assumption*, hence allowing terms to behave as flexible individual concepts across states. We study the complexity of formula satisfiability, showing that, without the rigid designator assumption, this problem for epistemic free DLs is NExpTime-complete (same as the logic $\mathbf{S5} \times \mathbf{S5}$ [27]), whereas it becomes undecidable for temporal free DLs interpreted on linear time structures (while it is decidable without definite descriptions and with the RDA [27]).

Proof details and examples are provided in an extended version of this article [28].

2. Epistemic and Temporal Free Description Logics

The DL $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ is a modalised extension of the free DL $\mathcal{ALCO}_u^\iota$ [10, 11]. Let $\mathsf{N_C}$, $\mathsf{N_R}$ and $\mathsf{N_I}$ be countably infinite and pairwise disjoint sets of *concept names*, *role names*, and *individual names*, respectively. The $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ *concepts* and *formulas* are defined as:

$$C ::= A \mid \{\tau\} \mid \neg C \mid C \sqcap C \mid \exists r.C \mid \exists u.C \mid \Diamond C, \qquad \varphi ::= (\alpha) \mid \neg\varphi \mid \varphi \land \varphi \mid \Diamond\varphi,$$

where $\tau ::= a \mid \iota C$ is an $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ *term*, $a \in \mathsf{N_I}$, $A \in \mathsf{N_C}$, $r \in \mathsf{N_R}$, u is the *universal role* and α is an $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ *axiom*, denoting either a *concept inclusion* (*CI*) of the form $C \sqsubseteq D$, or an $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ *assertion* of the form $C(\tau)$ or $r(\tau_1, \tau_2)$, where C, D are concepts, $r \in \mathsf{N_R}$, and τ, τ_1, τ_2 are terms. A term of the form ιC is called a *definite description*, and a concept $\{\tau\}$ is a *(term) nominal*. All the usual syntactic abbreviations are assumed, such as those for the *box* operator, $\Box C = \neg\Diamond\neg C$, and for the *reflexive* versions, $\Diamond^+ C = C \sqcup \Diamond C$ and $\Box^+ C = C \sqcap \Box C$.

Given an *epistemic frame* $\mathfrak{F} = (W, \sim)$, with W being a non-empty set of *worlds* (or *states*) and $\sim \subseteq W \times W$ being an equivalence relation on W, a *partial epistemic interpretation* based on $\mathfrak{F}$ is a triple $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, where: $\mathfrak{F}$ is the frame of $\mathfrak{M}$; Δ is a non-empty set, called the *domain* of $\mathfrak{M}$ (we adopt the so-called *constant domain assumption* [27]); and $\mathcal{I}$ is a function associating with every $w \in W$ a *partial interpretation* $\mathcal{I}_w = (\Delta, \cdot^{\mathcal{I}_w})$ that maps every $A \in \mathsf{N_C}$ to a subset of Δ, every $r \in \mathsf{N_R}$ to a subset of $\Delta \times \Delta$, the universal role u to the set $\Delta \times \Delta$ itself, and every a in a *subset* of $\mathsf{N_I}$ to an element in Δ. In other words, every $\cdot^{\mathcal{I}_w}$ is a total function on $\mathsf{N_C} \cup \mathsf{N_R}$ and a *partial* function on $\mathsf{N_I}$. We say that $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$ is a *total epistemic interpretation* if every $\mathcal{I}_w$, with $w \in W$, is a *total* interpretation, meaning that $\cdot^{\mathcal{I}_w}$ is defined as above, except that it maps *every* $a \in \mathsf{N_I}$ to an element of Δ.

Given $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, with $\mathfrak{F} = (W, \sim)$, we say that $\mathfrak{M}$ satisfies the *rigid designator assumption* (*RDA*) if, for every individual name $a \in \mathsf{N_I}$ and every $w, v \in W$, the following condition holds: if $a^{\mathcal{I}_w}$ is defined, then $a^{\mathcal{I}_w} = a^{\mathcal{I}_v}$, i.e., a is a *rigid designator*. An individual name $a \in \mathsf{N_I}$ is said to *denote in* $\mathcal{I}_w$ if $a^{\mathcal{I}_w}$ is defined, and we say that it *denotes in* $\mathfrak{M}$ if a denotes in $\mathcal{I}_w$, for some $w \in W$. Moreover, a is called a *ghost in* $\mathfrak{M}$ if, for every $w \in W$, a does not denote in $\mathcal{I}_w$. Dropping the RDA is the most general assumption, since rigid designators can be enforced by the

CI, $\Diamond^+\{a\} \sqsubseteq \Box^+\{a\}$. Also, partial interpretations generalise the classical ones: an individual can be forced to denote at some state (i.e., not being a ghost) with the CI, $\top \sqsubseteq \Diamond^+\exists u.\{a\}$, and at all states by the formula, $\Box^+(\top \sqsubseteq \exists u.\{a\})$. Note that a ghost individual is vacuously rigid.

Given $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, with $\mathfrak{F} = (W, \sim)$, and a world $w \in W$, we define the *value* $\tau^{\mathcal{I}_w}$ of a term τ in w as $a^{\mathcal{I}_w}$, if $\tau = a$, and as follows, for $\tau = \iota C$: $(\iota C)^{\mathcal{I}_w} = d$, if $C^{\mathcal{I}_w} = \{d\}$, for some $d \in \Delta$; and undefined, otherwise. As for the *extension* of a concept C in w, $C^{\mathcal{I}_w}$ is as usual with the following additions:

$$(\Diamond C)^{\mathcal{I}_w} = \{d \in \Delta \mid \exists v \in W, w \sim v \colon d \in C^{\mathcal{I}_v}\}, \quad \{\tau\}^{\mathcal{I}_w} = \begin{cases} \{\tau^{\mathcal{I}_w}\}, & \text{if } \tau \text{ denotes in } \mathcal{I}_w, \\ \emptyset, & \text{otherwise,} \end{cases}$$

where a term τ is said to *denote* in $\mathcal{I}_w$ if $\tau^{\mathcal{I}_w}$ is defined. A concept C is *satisfied at w of $\mathfrak{M}$* if $C^{\mathcal{I}_w} \neq \emptyset$. An $\mathbf{S5}_{\mathcal{ALCO}^\iota_u}$ *formula φ is satisfied at w of $\mathfrak{M}$*, written $\mathfrak{M}, w \models \varphi$, when:

$$\mathfrak{M}, w \models C(\tau) \;\text{ iff }\; \tau \text{ denotes in } \mathcal{I}_w \text{ and } \tau^{\mathcal{I}_w} \in C^{\mathcal{I}_w},$$
$$\mathfrak{M}, w \models r(\tau_1, \tau_2) \;\text{ iff }\; \tau_1, \tau_2 \text{ denotes in } \mathcal{I}_w \text{ and } (\tau_1^{\mathcal{I}_w}, \tau_2^{\mathcal{I}_w}) \in r^{\mathcal{I}_w},$$
$$\mathfrak{M}, w \models C \sqsubseteq D \;\text{ iff }\; C^{\mathcal{I}_w} \subseteq D^{\mathcal{I}_w}, \quad \mathfrak{M}, w \models \Diamond\psi \;\text{ iff }\; \exists v \in W, w \sim v \colon \mathfrak{M}, v \models \psi,$$

together with the usual interpretation of Boolean operators. An $\mathbf{S5}_{\mathcal{ALCO}^\iota_u}$ formula φ is *satisfied in* $\mathfrak{M}$ if there exists a world w in $\mathfrak{M}$ such that $\mathfrak{M}, w \models \varphi$, and it is *partial* (*total*) *satisfiable* if there is a partial (total) modal interpretation $\mathfrak{M}$ such that φ is satisfied in $\mathfrak{M}$.

For the temporal DL $\mathbf{LTL}_{\mathcal{ALCO}^\iota_u}$, we build $\mathbf{LTL}_{\mathcal{ALCO}^\iota_u}$ *terms*, *concepts*, and *formulas* similarly to the $\mathbf{S5}_{\mathcal{ALCO}^\iota_u}$ case, by using the temporal operator *until*, $\mathcal{U}$, for the construction of concepts, $C \,\mathcal{U}\, D$, and formulas, $\varphi \,\mathcal{U}\, \psi$. $\mathbf{LTL}_{\mathcal{ALCO}_u}$ is obtained by disallowing descriptions. The *flow of time* is $\mathfrak{F} = (\mathbb{N}, <)$, where $\mathbb{N}$ is the set of natural number and $<$ is the linear order on $\mathbb{N}$. A *partial temporal interpretation*, or *partial trace*, based on $\mathfrak{F}$, is a triple $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, defined as in the epistemic case. We similarly define the notion of *total trace*. Given a partial trace $\mathfrak{M} = (\mathfrak{F}, \Delta, \mathcal{I})$, with $\mathfrak{F} = (\mathbb{N}, <)$ and $t \in \mathbb{N}$ (that we call an *instant of* $\mathfrak{M}$), the *value* of an $\mathbf{LTL}_{\mathcal{ALCO}^\iota_u}$ term τ at t, the *extension* of an $\mathbf{LTL}_{\mathcal{ALCO}^\iota_u}$ concept C at t, the *satisfaction* of a $\mathbf{LTL}_{\mathcal{ALCO}^\iota_u}$ formula φ at t, are defined as for the modal case, by replacing the semantics of the $\Diamond$ modal operator with the following one for the $\mathcal{U}$ temporal operator:

$$(C \,\mathcal{U}\, D)^{\mathcal{I}_t} = \{d \in \Delta \mid \text{there is } u \in T, t < u \colon d \in D^{\mathcal{I}_u} \text{ and, for all } v \in (t, u), d \in C^{\mathcal{I}_v}\},$$
$$\mathfrak{M}, t \models \varphi \,\mathcal{U}\, \psi \text{ iff there is } u \in T, t < u \colon \mathfrak{M}, u \models \psi \text{ and, for all } v \in (t, u), \mathfrak{M}, v \models \varphi.$$

An $\mathbf{LTL}_{\mathcal{ALCO}^\iota_u}$ formula φ (respectively, a concept C) is (*partial* or *total*) *satisfiable*, respectively, if φ (respectively, C) is satisfied at instant 0 in some (partial or total) trace $\mathfrak{M}$, respectively.

Assertions are syntactic sugar, since $C(\tau)$ and $r(\tau_1, \tau_2)$ are captured by the following CIs, respectively: $\top \sqsubseteq \exists u.\{\tau\}, \{\tau\} \sqsubseteq C$; and $\top \sqsubseteq \exists u.\{\tau_1\}, \{\tau_1\} \sqsubseteq \exists r.\{\tau_2\}$. To avoid ambiguities, we use parentheses when applying Boolean or modal operators to assertions. Thus, for instance, the formulas $\neg(C(\tau))$ and $\Diamond(C(\tau))$ abbreviate, respectively, $\neg(\top \sqsubseteq \exists u.\{\tau\} \wedge \{\tau\} \sqsubseteq C)$ and $\Diamond(\top \sqsubseteq \exists u.\{\tau\} \wedge \{\tau\} \sqsubseteq C)$, whereas the assertions $\neg C(\tau)$ and $\Diamond C(\tau)$ stand, respectively, for $\top \sqsubseteq \exists u.\{\tau\} \wedge \{\tau\} \sqsubseteq \neg C$ and $\top \sqsubseteq \exists u.\{\tau\} \wedge \{\tau\} \sqsubseteq \Diamond C$. Finally, as already observed for $\mathcal{ALCO}^\iota_u$ [11], we point out that formulas are just syntactic sugar in $\mathcal{ML}_{\mathcal{ALCO}^\iota_u}$, since a CI $C \sqsubseteq D$ can be internalised [29, 30] as a concept of the form $\forall u.(C \Rightarrow D)$.

We also remark on a counter-intuitive behaviour without the RDA assumption. Let us consider the following formula: $(\{a\} \sqsubseteq \Box C) \wedge \Diamond(\{a\} \sqsubseteq \neg C)$. This formula, while unsatisfiable if the RDA is assumed, is satisfiable without the RDA, since it is satisfied in an epistemic or temporal interpretation that interprets the individual name a differently in different states.

On $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ satisfiability, we show the following, by adapting a *quasimodel* technique [27] to cover the case of possibly uninterpreted individual names not constrained by the RDA.

Theorem 1. *Partial $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$ formula satisfiability without the RDA is NExpTime-complete.*

Concerning $\mathbf{LTL}_{\mathcal{ALCO}_u^\iota}$ satisfiability, we have the following negative results, the proof of which is based on Degtyarev et al. [31] (related results appear also in Hampson and Kurucz [32]).

Theorem 2. *Formula satisfiability in $\mathbf{LTL}_{\mathcal{ALCO}_u}$ without the RDA, and in $\mathbf{LTL}_{\mathcal{ALCO}_u^\iota}$ with RDA, is undecidable.*

Undecidability holds already for *total* satisfiability. Similar results apply also to $\mathbf{LTL}^{\boldsymbol{f}}_{\mathcal{ALCO}_u^\iota}$, interpreted on *finite* traces (using the standard translation of temporal DLs into temporal first-order logic [27], and the reduction of the latter satisfiability from finite to infinite traces [33]).

3. Discussion and Future Work

We conducted a preliminary study on modal free description logics, in particular on the epistemic free DL $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$, and on the temporal free DL $\mathbf{LTL}_{\mathcal{ALCO}_u^\iota}$. Syntactically, these DLs extend the classical $\mathcal{ALCO}_u$, with nominals and the universal role, by including definite descriptions and epistemic or temporal operators. Semantically, we interpret these DLs over modal interpretations that allow for *non-denoting* terms and *non-rigid* designators. We show that, while formula satisfiability is NExpTime-complete for $\mathbf{S5}_{\mathcal{ALCO}_u^\iota}$, it becomes undecidable for $\mathbf{LTL}_{\mathcal{ALCO}_u^\iota}$.

On the epistemic side, as future work we plan to: (i) consider frames for the propositional modal logics $\mathbf{K4}$, $\mathbf{T}$, $\mathbf{S4}$, or $\mathbf{KD45}$, to model different doxastic or epistemic attitudes [27]; (ii) investigate non-rigid descriptions and names in the context of *non-normal* modal DLs [34, 35, 36], to avoid the *logical omniscience* problem (i.e., an agent knows all the logical truths and all the consequences of their background knowledge), which affects all the systems extending $\mathbf{K}$ [37, 38]; (iii) address less expressive DL languages, such as $\mathcal{ELO}_u^\iota$, in an epistemic setting, and connect them with the recently investigated *standpoint DL* family [39, 40].

On the temporal side, we believe that the negative results presented here do not entirely undermine the use of definite descriptions on a temporal dimension. For applications in temporal conceptual modelling and ontology-mediated query answering [41, 42], it is worth exploring whether more encouraging results can be obtained in fragments restricting the use of temporal operators (limited, e.g., to the $\Box$ operator only), or constraining the DL dimension (as in $\mathbf{LTL}_{\mathcal{ALCO}^\iota}$, without the universal role, or in the *TDL-Lite* family [43]).

Finally, we are interested in studying in this setting the complexities of other problems than formula satisfiability. Related to *interpolant* and *explicit definition existence* [44, 45, 46], the *referring expression existence* problem [11], i.e., deciding the existence of an individual's description given a signature and an ontology, is of particular interest to our modal free DLs.

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