Solving Vector Optimization Problems on Combinatorial Configurations With Fuzzily Specified Data

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Abstract

The paper presents the formulation of the vector optimization problem on the combinatorial configuration of permutations with fuzzily specified data of the vector functions of the criteria and the feasible domain. The properties of the set of feasible solutions of the given problem are described. To solve the formulated problem, two approaches based on the method of guaranteed result and the method of successive concessions are proposed. Methods of solving multi-criteria problems with vague input information are presented. The main advantages of using new models are that they are linear, can generate different solutions of vector (multi-criteria) problems by changing the threshold values and set tolerance limits of fuzzy goals. There are lots of fuzzy data in the real world, and these data should be used in intelligent systems. One can find successful fuzzy systems in almost all industrial areas where optimization, learning, and handling imprecise knowledge play a role, i.e. classification, prediction, planning, control, and decision-making – just to mention a few fruitful areas. Fuzzy rule-based models often turn out to be helpful, understandable, not complex, and easy to handle.

Keywords [1]

vector optimization problems; combinatorial configuration of permutations; fuzzy sets, Pareto set.

1. Introduction

In the decision-making process, situations often arise that have one or another degree of uncertainty, and therefore the quality of problem solving depends on the complete consideration of all factors affecting their consequences. Often these factors are subjective, and this applies as a decision maker, as well as the decision-making process itself. In addition, the decision maker does not always have at his disposal all the information necessary for his justified actions. This is one of the main difficulties that arise in the decision-making process. Such situations reflect the insufficiency of information for setting the problem, therefore, under unclear conditions and criteria, decision-making becomes problematic. When modeling real problems, vagueness appears, in particular, in the form of a description of functions and parameters on which they depend [1]. A convenient mathematical tool used to describe and take into account such information is the theory of fuzzy sets, first proposed in [2] and described, in particular, in [3-4]. Fuzzy sets are widely used in various applications of artificial intelligence, the theory of pattern recognition, decision-making, etc. [4-8].

In many theoretical and practical problems, there is a need to make a decision taking into account several optimality criteria [9-12]. At the same time, multi-criteria optimization problems are quite common in practice, in which a finite set of alternatives (solutions) are specified, which can be evaluated both quantitatively and qualitatively [12, 13]. The peculiarity of such problems, as a method of mathematical modeling of various applied problems, is that the multi-criteria selection of the most

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appropriate solution is carried out from a set of unimproved solutions. The Pareto principle plays an exceptional role in solving such problems, according to which the optimal solution should be chosen among the Pareto-optimal solutions forming the compromise area. Note that this principle is not universal and applies only when a number of conditions are met. Even if these conditions are met, constructing a set of Pareto-optimal solutions can cause significant difficulties [14].

Another approach to solving the problem of multi-criteria optimization is the idea of successive concessions, based on the ranking of criteria in order of decreasing importance and solving a single-criteria optimization problem, in which the most important criterion takes an extreme value, and restrictions are imposed on the others. The disadvantage of this approach is the complication of the conditions of the input problem, namely the limitations of the admissible area and the need to analyze different variants of the problem. The transition to a single-criteria problem is possible by aggregating individual criteria into a generalized criterion using the appropriate convolution [8-10].

Despite the external attractiveness of such an approach, it raises a number of questions: it is not clear how to determine the type of aggregation function; it is difficult or impossible to justify the principle of evaluating its parameters, in particular weighting factors, degree indicators, as well as problematic interpretation of the obtained results [11-12]. Therefore, the problem of finding a set of Pareto-optimal solutions of the vector optimization problem is of great practical and theoretical importance. It should be noted that in most applied problems, the formal formulation of the optimization of a vector mathematical model is not only difficult, but also in a number of cases the main parameters may be vaguely specified. Models and methods of fuzzy optimization are used in economics, management, medicine, multi-objective planning, when solving operations research problems, in transport systems. Vaguely specified data in such models can be both in the description of the objective functions of the problem and its admissible area [7, 15].

Decision-making methods based on fuzzy models allow for convenient and high-quality evaluation of alternatives according to individual criteria. Unlike other methods, adding new alternatives does not change the order of previously ranked sets. When evaluating alternatives according to criteria, both linguistic evaluation and evaluation based on point evaluations using membership functions are possible. The main problem of multi-criteria selection using fuzzy models is providing information about the relationship between criteria and methods of calculating integral estimates. Methods based on different approaches give different results. Each approach has its limitations and features. The study of the problem of decision-making in a fuzzy environment became possible thanks to the publication of the article by R. Bellman and L. Zadeh [1].

The present paper continues researches, presented in works [7, 12]. This paper formulates the formulation of the vector optimization problem on the combinatorial configuration of permutations as a problem with vaguely specified data. Thus, the Edgeworth–Pareto principle extends to a wider class of multi-criteria problems in which the set of admissible solutions is fuzzy or the objective function has fuzzy parameters. In works, in particular, [15−19] investigated problems with many criteria with fuzzy objective functions, and in [20−28] - problems on combinatorial sets. Obviously, it is expedient to consider problems combining the above.

2. Preliminaries

Fuzzy subsets are formed by introducing the generalized concept of belonging, i.e., the expansion of the two-element set of values of the characteristic function to the continuum.

This means that the transition from full membership of an object to full non-belonging occurs smoothly, not in a jump, so the membership is expressed by a number from the interval, and not by one of the two values of the elements of the set, as in the case of indicators of ordinary subsets.

Regardless of whether fuzzy or clear subsets are used, the determination of degrees of belonging relies on some subjective decision maker criteria. In some cases, the determination of the corresponding values of the degrees of belonging of the elements of fuzzy sets leads to significant difficulties in working with fuzzy concepts. Formally, the general problem of fuzzy mathematical programming is described in the following way [15].

Let X is a universal set of alternatives, $\mu_A : X \to [0, 1]$ is a given fuzzy subset of feasible alternatives, Y is a universal set of evaluations of the results of choices of alternatives from the set

X , and $\mu_R : Y \times Y \to [0, 1]$ is a given fuzzy preference ratio on the set.

Choices of alternatives are evaluated by fuzzy values of a given fuzzy objective function $\varphi: X \times Y \to [0, 1]$. The task is to make a rational choice of alternatives based on the information given in the form described above. The next step on the way to refine the model considered here is to describe the parameters of the problem in the form of fuzzy sets. At the same time, in addition to specifying sets of possible parameter values, additional information is introduced into the model in the form of membership functions of these fuzzy sets. These functions can be considered as a method of an approximate display by an expert in an aggregated form of his informal idea about the real value of a given parameter. The values of the membership function are the weights that the expert assigns to the different possible values of this parameter. There is no doubt that taking into account such additional information complicates the input mathematical model.

For further exposition, we define a generalization of the concepts of multiset, n-sample, and combinatorial set of permutations for the case of vaguely specified information.

Definition 1 [7, 12, 28]. A fuzzy multiset $\tilde{X}$ defined on a universal multiset X is a set of pairs $\left(x, \mu_{\tilde{X}}(x)\right)$, where $x \in X$, $\mu_{\tilde{X}}(x)$ the function, $\mu_{\tilde{X}}(x)$: $X \to [0,1]$, is called the membership function of the multiset $\tilde{X}$.

The value $\mu_{\tilde{X}}(x)$ for a particular x is called the degree of belonging of this element to the fuzzy multiset $\tilde{X}$.

As you know, multisets, according to the definition, form a subclass of the class of fuzzy multisets. A number of operations are performed on fuzzy sets, as well as on classical sets, such as union, intersection, Cartesian product, difference, etc.

These operations also apply to fuzzy multisets [7].

Let be a given fuzzy multiset

$\tilde{A} = \left\{\left(a_1, \mu_{\tilde{A}}(a_1)\right), \left(a_2, \mu_{\tilde{A}}(a_2)\right), ..., \left(a_q, \mu_{\tilde{A}}(a_q)\right)\right\}$, its basis

$S(\tilde{A}) = \left\{\left(e_1, \mu_{\tilde{A}}(e_1)\right), \left(e_2, \mu_{\tilde{A}}(e_2)\right), ..., \left(e_k, \mu_{\tilde{A}}(e_k)\right)\right\}$, where

$\mu_{\tilde{A}}(e_i) = \min\left\{\mu_{\tilde{A}}\left(a_{i_j}\right) \middle| a_{i_j} = a_{i_t}, j \neq t, \forall i, j, t \in N_q\right\}$,

$e_j \in R_1 \ \forall j \in N_k = \{1, ..., k\}$ and

multiplicity of elements $k(e_j) = r_j$, $j \in N_k$, $r_1 + r_2 + ... + r_k = q$.

An ordered fuzzy n-sample from a fuzzy multiset $\tilde{A}$ is called a set

$$a = \left(\left(a_{i_1}, \mu_{\tilde{A}}(a_{i_2})\right), \left(a_{i_2}, \mu_{\tilde{A}}(a_{i_2})\right), \ldots, a_{i_n}, \mu_{\tilde{A}}(a_{i_n})\right), \tag{1}$$

where $a_{i_j} \in \tilde{A} \ \ \forall i_j \in N_k$, $\forall j \in N_k$, $i_s \neq i_t$, if $s \neq t \ \ \forall s \in N_k$, $\forall t \in N_k$.

Definition 2. [7] A fuzzy subset $P(\tilde{A})$ whose elements are fuzzy n-samples of the form (1) from a fuzzy multiset $\tilde{A}$ is called a fuzzy Euclidean combinatorial set if the following conditions are satisfied for an arbitrary pair of its elements

$$a = \left(a_1, \mu_{\tilde{A}}(a_1)\right), \left(a_2, \mu_{\tilde{A}}(a_2)\right), ..., \left(a_n, \mu_{\tilde{A}}(a_n)\right) \text{ and}$$

$$b = \left(b_1, \mu_{\tilde{A}}(b_1)\right), \left(b_2, \mu_{\tilde{A}}(b_2)\right), ..., \left(b_n, \mu_{\tilde{A}}(b_n)\right):$$

$$(a \neq b) \Leftrightarrow (\exists j \in N_n : \left(a_j, \mu_{\tilde{A}}(a_j)\right) \neq \left(b_j, \mu_{\tilde{A}}(b_j)\right),$$

that is, a set $P(\tilde{A})$ has the following property: two elements of a set $P(\tilde{A})$ are different from each other if, regardless of other differences, they differ in the order of placement of the symbols that make them up and in the degree of belonging to a fuzzy subset $P(\tilde{A})$.

A fuzzy set of permutations with repetitions of n real numbers, among which k are different, is called a general fuzzy set of permutations and is denoted by $P_{nk}(\tilde{A})$.

Definition 3. [7] A convex combination of fuzzy sets $A_1, A_2, ..., A_n$ in R^n is called a fuzzy set A with a membership function of the form

$$\mu_A(x) = \sum_{i=1}^{n} \lambda_i \mu_i(x), \text{ where } \lambda_i \geq 0,\ i \in N_n, \sum_{i=1}^{n} \lambda_i = 1.$$

We will consider the elements of the set of permutations with repetitions as points of the arithmetic Euclidean space R^n.

It is known that each element of the set $P_{nk}(\tilde{A})$ is an ordered set of n real numbers, among which k are different. Without losing generality, we arrange the elements of the set A as follows:

$$a_1 \leq a_2 \leq \ldots \leq a_n. \tag{2}$$

As is known [29−31], the convex hull of a set of permutations in Euclidean space is a polyhedron of permutations, the set of vertices of which coincides with the set of permutations.

A permutation polytope of order n is an $(n-1)$-dimensional convex polytope embedded in an n-dimensional Euclidean space that is the convex hull of all n! points obtained by permuting the coordinates of the vector $(1, 2, ..., n)$. According to Ziegler, Günther [29], the permutation polyhedron began to be realized in the works of Schute in 1911 [30].

The term "permutation polyhedron" itself (more precisely, its French version "permutoèdre") first appeared in an article by Guibaud G.-T and Rosenstahl P. in 1963. Bowman V.-J. in 1972 in a more general situation, used the term "permutation polytope" for any polytope whose vertices are in one-to-one correspondence with permutations of some set [31].

Along with the classical permutation polyhedron, we describe the general permutation polyhedron $\Pi_{nk}(\tilde{A})$, which is the convex hull of the general set of permutations $P_{nk}(\tilde{A})$ [7, 12]:

$$\sum_{j=1}^{n} x_j \leq \sum_{j=1}^{n} a_j,\ \sum_{j=1}^{i} x_{\alpha_j} \geq \sum_{j=1}^{i} a_j, \tag{3}$$

$\alpha_j \in N_n,\ \ \alpha_j \neq \alpha_t,\ \ \forall j \neq t,\ \ \forall j,\ t \in N_i,\ \ \forall i \in N_n,\ P_{nk}(A) = \text{vert}\,\Pi_{nk}(A)$.

A fuzzy convex polyhedron can also be represented as a convex hull of a fuzzy combinatorial set of permutations:

$$\Pi_{nk}(\tilde{A}) = \text{conv}\, P_{nk}(\tilde{A}).$$

3. Formulation of the vector optimization problem on the combinatorial configuration of permutations with fuzzy specified data

The vector problem of combinatorial optimization is considered

$$Z(F, X): \max\left\{F(x) \mid x \in X \subset R^n\right\},$$

$$F(x) = (f_1(x), ..., f_l(x)),$$

$$f_i: R^n \to R,\ i \in N_l,$$

$$X = \text{vert}\,\Pi_{nk}(A) \cap D \neq \varnothing,\ \Pi_{nk}(A) = \text{conv}\, P_{nk}(A),$$

where $P_{nk}(A)$ − combinatorial set of permutations, $D \subset R^n$ − convex polyhedron.

A fuzzy subset $\tilde{X} = \{x, \mu_{\tilde{X}}(x)\}$, is given on the set X.

$\tilde{X} = \{x, \mu_{\tilde{X}}(x)\}$, where $x \in X$, and $\mu_{\tilde{X}}(x): X \to [0,1]$ − set membership function $\tilde{X}$.

By maximization we mean the selection of a fuzzy subset $\tilde{R}$ from a fuzzy set $\tilde{X}$, which corresponds to the largest value, as a vector function F, and membership functions $\mu_{\tilde{X}}(x)$ of a fuzzy set of alternatives. These alternatives in multicriteria optimization problems are called efficient (Pareto optimal). An interesting case is when the vector optimization problem is a problem with a fuzzy-defined vector objective function.

A fuzzy decision making problem defined over a feasible set X of decision variable vectors assumes the existence of several fuzzy goals $G_k, k=1,...,l,$ that are fuzzy subsets of X under a set of fuzzy restrictions $R_i, i=1,...,m,$ that are also fuzzy subsets of X. Bellman and Zadeh [1] described a solution to such problem (i.e. a decision), through a fuzzy subset of X, i.e. a set $\{x, \mu D(x) \mid x \in X\}$, where the membership function

$\mu_D : X \to [0,1]$ is defined by aggregating the fuzzy goals and restrictions using the min operator

$$\mu_D(x) = \min\left(\left\{\mu_{G_k}(x) \mid k = 1,...,l\right\} \cup \left\{\mu_{R_i}(x) \mid i = 1,...,m\right\}\right).$$

The classic way to construct a fuzzy goal related to any kind of objective functions f_i, that has to be maximized is to involve a threshold (g_i) and a tolerated limit $(t_i < g_i)$ on the given threshold, and define the membership function $\mu_{f_i}(f_i(x))$ $\mu_{\tilde{X}}(f_i(x))$, where

$$\mu_{f_i}(f_i(x)) = \begin{cases} 0, \ f_i(x) < t_i, \\ 1 - \dfrac{f_i(x) - t_i}{g_i - t_i}, t_i \le f_i(x) \le g_i, \ i \in N_l = \{1,...,l\}. \\ 1, \ f_i(x) > g_i. \end{cases}$$

Due to the established inequality between the threshold g_i and the tolerance limit t_i, $\mu_{f_i}(f_i(x))$ is a component-wise increasing function. The greater the degree of belonging of the alternative x to the fuzzy set of the goal, that is, the greater the value of the function $\mu_{f_i}(f_i(x))$, the higher the degree of achievement of this goal will be if alternative *x* is chosen as a solution *x*.

Further on, Zimmermann [4] proposed the following mathematical problem

$$\begin{gathered} \max \alpha \\ \mu_{G_k}(x) \ge \alpha, \ k = 1,...,l, \\ \mu_{R_i}(x) \ge 0, \ i = 1,...,m, \\ 0 \le \alpha \le 1, \ x \in X, \end{gathered}$$

to derive the optimal decision, namely the solution with the maximal membership value.

The solution of a fuzzy multicriteria optimization problem can be reduced to the solution of a crisp problem by transforming the constraints into the form

$$\alpha \to \max, \mu_k(f_k(x)) \ge \alpha, \ x \in X,$$

where $\alpha -$ level (cut) of a fuzzy set $\tilde{X}$.

4. Approaches to solving the vector optimization problem on combinatorial configurations with fuzzy specified data

There are quite a few methods for solving multicriteria problems, but most of them are designed to solve problems of choosing solutions in a well-defined space. A small modification makes them applicable even in conditions of fuzzy.

In particular, in practice, classical optimization theory is often applied to fuzzy models, where there is no reason to set coefficients in the form of precisely defined numbers because such an artificial narrowing of a priori information can lead to distortion of the obtained results.

4.1. Problem solving based on the guaranteed result method

To solve the problem formulated above, the guaranteed result method is considered, which gives a good result even for the smallest of the criteria, i.e., a compromise solution is obtained by solving the following optimization problem:

$$z = \min_{i=1,2,...l} f_i(x) \to \max, \ x \in X.$$

As you know, taking into account the normalization of criteria, methods of guaranteed results are the most promising direction in solving multi-criteria optimization problems.

For normalized criteria

$$\lambda_k(x) = \frac{f_k(x)}{f_k^*} : R^n \to R, \ k \in N_l,$$

where $f_k^* = \max_{x \in X} f_k(x) : R^n \to R, \ k \in N_l$, the maximin problem is formulated in the form:

$$z = \min_{k \in N_l} \lambda_k(x) \to \max : x \in X, R^n \to R, \ k \in N_l. \tag{4}$$

Let us consider two cases when the criteria are equal and unequal (with a given priority).

Consider the case when the criteria are equivalent.

Problem (4) is equivalent to problem

$$z=\lambda \to \max \tag{5}$$

under conditions

$$\begin{cases} \lambda \le \lambda_k(x), k \in N_l, \\ x \in X, \end{cases} \tag{6}$$

$$X = \operatorname{vert} \Pi_{nk}(A) \cap D \ne \varnothing, \ \Pi_{nk}(A) = \operatorname{conv} P_{nk}(A),$$

where $P_{nk}(A)$ − combinatorial set of permutations, $D \subset R^n$ − convex polyhedral set.

Problem (5) − (6) is called a λ-problem. It has a linear objective function and $m+l$ constraints.

If all functions $f_k(x), k \in N_l$, $g_i(x), i \in N_m$ are linear, then the λ-problem belongs to linear programming. In this case, it is proved that the optimal solution x^* of the λ-problem is Pareto optimal.

Consider the case when the priority of the criteria is set. Let there be two criteria $f_1(x)$ and $f_2(x)$, and $\lambda_1(x)$ and $\lambda_2(x)$ − are the corresponding normalized criteria. Let's divide the feasible region into two parts $X = X_1 \cup X_2$ in such a way that the inequality $\lambda_1(x) > \lambda_2(x)$ is satisfied in the region X_1, that is, the first criterion has priority over the second, and in the region X_2 the inequality $\lambda_1(x) \le \lambda_2(x)$ is satisfied, the second criterion has priority over the first.

For the numerical characteristic of the priority, the connection coefficient is introduced $p(x) : \lambda_1(x) = p(x)\lambda_2(x)$, which determines how many times the relative estimate $\lambda_1(x)$ is greater than $\lambda_2(x)$. If x^* is an optimal point for equivalent criteria, then $p(x^*) = 1$.

If x_1^* is the optimum point according to the first criterion, where $\lambda_1(x_1^*) = 1$, $\lambda_2(x_1^*) < 1$, that is $x_1^* \in X_1$, and it means that $p(x_1^*) > 1$.

Similarly, if x_2^* is the optimum point according to the 2nd criterion, where $\lambda_1(x_2^*) < 1$, $\lambda_2(x_2^*) = 1$, then it means that $p(x_2^*) < 1$.

Let the first criterion have priority over the second. Then the coefficient $p(x)$ must be set in the interval $(1; p(x_1^*))$, and then the λ-problem must be formulated and solved, including the equality in the system of constraints

$$\lambda_1(x) = p(x)\lambda_2(x).$$

As a result, we will get the point x^*, which will belong to the set X_1, where the first criterion has priority over the second. t is proved that for convex problems of multicriteria optimization, the point x^*, which is the solution of the λ-problem, is unique and Pareto optimal. The disadvantage of the considered method is the subjectivity of setting the connection coefficient $p(x)$.

Solving the problem of multicriteria optimization by the method of a guaranteed result, as a rule, goes through the following stages:

1. Development of a mathematical model of the system based on set goals and limitations; at the same time, the opinion of experts is often used.
2. Preliminary analysis of the system separately for each partial criterion; use methods and software tools of single-criteria optimization.
3. Standardization of criteria.
4. Solving the multicriteria optimization problem with equivalent criteria.
5. Determining the priorities of the criteria and solving the multi-criteria optimization problem with assigned priorities.

4.2. Approach to solving the problem based on the method of successive concessions

The development of methods for solving the given problem in conditions of vague certainty requires knowledge and use of the results of the operations of finding the sum, product, minimum and maximum of vague values.

By a fuzzy number, we will understand a fuzzy set with a definition area in the form of an interval of the real axis R^1. We denote the set of all fuzzy numbers $\tilde{R}^1$ defined by R^1.

Let x and y be two fuzzy numbers with carriers $S_x = (a_1, a_2)$ and $S_y = (a_1, a_2)$, respectively:

$$a_2 > a_1, b_2 > b_1 \;;$$

$$g: R^1 \times R^1 \to R^1 \text{ - some function.}$$

Then, according to the principle of generalization, the fuzzy number is determined by the membership function

$$\mu_D(z) = \sup_{\substack{g(a,b)=z \\ a\in S_x,\;\; b\in S_y}} \min\{\mu_x(a), \mu_y(b)\} \tag{7}$$

Let us denote $\otimes -$ one of the four arithmetic operations: +, −, $\cdot$, /; $g(a,b) = a \otimes b$. Then formula (7) determines the result of an arithmetic operation $\otimes$ on fuzzy numbers x and y.

If $g(\cdot)-$ is a function of not two, but n arguments, then the principle of generalization is formulated analogously to formula (7). When comparing two vague values, it is necessary to define the equality of these values.

Definition 4. Two fuzzy values (two numbers) $(x_1, \mu_1(x_1))$ and $(x_2, \mu_2(x_2))$ we will consider equal if $x_1 = x_2$ and $\mu_1(x_1) = \mu_2(x_2)$.

Definition 5. If the condition $x_1 \geq x_2$, $\mu_1(x_1) \geq \mu_2(x_2)$ and one of these inequalities is strict, then the fuzzy quantity $(x_1, \mu_1(x_1))$ is greater than the fuzzy quantity $(x_2, \mu_2(x_2))$.

An approach based on the method of successive concessions has been developed. When solving a multi-criteria problem by the method of successive concessions, a qualitative analysis of the relative importance of partial criteria is first made.

The peculiarity of this method is that the problem criteria must be pre-numbered in descending order of their importance, thus the main criterion $f_1(x)$ is less important than $f_2(x)$, followed by other partial criteria $f_3(x)$, $f_4(x)$,…, $f_l(x)$. The most important criterion is maximized $f_1(x)$ and its largest value is determined f_1^*. Then the value of the permissible reduction (concession) $\Delta_1 \geq 0$ of the criterion $f_1(x)$ is assigned and the largest value f_2^* of the second criterion is found $f_2(x)$, provided that the value of the first criterion must not be less than $f_1^* - \Delta_1$.

The amount of the concession is again assigned $\Delta_2 \geq 0$, but according to the second criterion, which is used together with the first when finding the conditional maximum of the third criterion, etc. Finally, the last most important criterion is maximized $f_l(x)$, provided that the value of each criterion $f_r(x)$ from $l-1$ the previous ones must be no less than the corresponding value $f_r^* - \Delta_r$, then the solutions obtained as a result are considered optimal.

Thus, the choice of the solution of the problem is carried out by performing a multi-step procedure and consists in sequentially including the constraints of the problem $Z(F, X)$ and taking into account the structural features of its admissible area.

The optimal solution is considered to be the solution of the last problem from the following sequence of problems:

$$f_1^* = \max\left\{ f_1(x) \middle| x \in X \right\},$$

$$f_2^* = \max\left\{ f_2(x) \middle| x \in X,\, f_1(x) \geq f_1^* - \Delta_1 \right\}, \ldots,$$

$$f_l^* = \max\left\{ f_l(x) \middle| x \in X,\, f_{r-1}(x) \geq f_{r-1}^* - \Delta_{r-1}, r \in N_l \right\}.$$

It should be noted that in the case when all Δ_r are zero, the method of successive concessions selects only lexicographically optimal strategies; these strategies deliver the largest solution to the most important criterion in the set of admissible values $f_1(x)$. Therefore, the amount of concessions intended for a multi-criteria task can be considered as a kind of measure of deviation of the priority (degree of relative importance) of partial criteria from the rigid, lexicographic one.

The concept of structures of dominance and non-dominated solutions in multi-criteria problems allows us to consider general cases in which there is information about the preferences of the decision-maker. In [16], the concepts of fuzzy convex and fuzzy polar cones are introduced, which generalize the structures that will be used to define the concepts of optimality according to Pareto, Slater, and Smale. If there is no information about both the preferences for a set of alternatives and the preferences for a set of criteria, then as a rule, the simplest methods are used: minimax, maximax, etc. If there is information only about the comparative importance of evaluations according to each of the criteria, they use methods of sequential consideration of alternatives according to individual criteria (lexicographic method, method of permutations, method of sequential reduction, etc.). If the decision-maker's preferences on sets of criterion evaluations are expressed in ordinal scales and set in relation to the weight of the criteria, then voting methods are used, the most common of which in decision-making is the B. Roux method. If the relative weights of the criteria and the relative values of the criterion evaluations for individual criteria can be obtained, then many different methods are used, in particular, direct methods of evaluating alternatives using predetermined evaluation functions (for example, additive weighted convolution of evaluations for all criteria), utility theory methods that require dialogue with the decision-maker and submission to his known axiomatics.

If along with the information about the importance of the criteria, the ideal criterion evaluations are known, it is possible to apply methods for evaluating the achievement of goals.

5. Conclusions

The paper presents the formulation of the vector optimization problem on the combinatorial configuration of permutations with vaguely specified data. The vagueness is specified in the description of the objective function and the admissible domain of the problem. The Edgeworth–Pareto principle applies to a class of multicriteria problems in which the set of possible solutions is fuzzy, or the objective function has fuzzy parameters.

Methods of solving multi-criteria problems with vague input information are presented. Depending on the specifics of the task, it is possible to apply other methods of multicriteria selection modified in case of vaguely specified information. The generalization of clear methods, as a rule, does not present particular difficulties, if the methods of presenting vague concepts, implementing vague calculations, comparing vague numbers, and forming a vague set of better alternatives are chosen in accordance with the conditions of the problem being solved.

As a result of the research of the vector combinatorial problem, which is based on the use of information about the convex hull of the admissible domain, the study of the properties of the polyhedron, the vertices of which are defined by a vaguely specified combinatorial set of permutations, a method of solving complex multicriteria problems on the specified combinatorial set was developed and substantiated. In the coming papers, we plan to investigate more special vector models on various combinatorial configurations with vaguely specified data, to develop new versions of algorithms for solving the specified problems. The construction of randomized versions of algorithms is also of considerable interest.

The obtained results are important and relevant, as they can be applied in the functioning of complex real systems, for example, economic, ecological and a number of other artificial and natural systems. They can have a continuation for the development of new fuzzy optimization models on various combinatorial configurations and methods for solving vector optimization problems using the concepts of fuzzy combinatorial objects and be used, including for the construction of computer technologies with the organization of intelligent calculations when solving complex decision-making problems.

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